OCEAN EXISTS BEHIND THE WAVES: THE LAW OF LOGIC VS THE LOGIC OF LAW

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Abstract:

The intention behind the writing this brief digest themed on "OCEAN EXISTS BEHIND THE WAVES" is simply to centralize the ideology of three typically different communities, namely Poets, Vedic Sheers and Mathematicians through a common frame of reference "THE LAW OF LOGIC VS THE LOGIC OF LAW". More specifically, the present digest is meant to excavate the analogy among the inference about the notion of **'infinity'**, the law of nature called **'completeness'** and another law of nature called **'unity or connectedness'**.

Keywords: Completeness, Connectedness, logic, law, infinity, Bhagwat Gita

1. INTRODUCTION:

Before I take you to the journey of "OCEAN EXISTS BEHIND THE WAVES", let me

nourish your with few rhythmsⁱ, which may help you discarding the mirage like 'I', 'me', 'mine' i.e., the false ego evolved by our natural instinct, which the ancient seersⁱⁱ (Rishis, Munis) of the six Indian Philosophical system, especially the Samkhya wisdom seekers enunciated as 'Ahamkaar'.

Don't you feel, these rhythms are penetrating your whole hallucination accumulated on the mirror of your mind? Anyways, these rhythmic patterns have a lot to reveal. The connotation of these lines, to me as a Mathematics lover is far more transcendental. These are not merely the rhythmic lines to me, but are much more lawlike e.g., 'the axioms' coiled with perpetual logics. I am mesmerized to see that what an axiom of choice is used here!

May my Existence be crushed; may this turn out to be corpuscular. Let me taste the 'taste of completeness''. Please bestow me the rainbows of existence filled with vibrant celebration.... Oh, Aadi Yogi 'Oh Eternal Existence' Oh "Master of connectedness".

For me, this kind of rhythmic pattern of words seems to be logic of natural law having capabilities of moulding the fabric of our intellect towards the truth, i.e., law of logic.

The rhythms are...

"पीस दो अस्तित्व मेरा,और कर दो चूरा चूरा । पूर्ण होने दो मुझे और होने दो अब पूरा पूरा। भस्म वाली रस्म कर दो आदि योगी। योग उत्सव रंग भर दो आदि योगी"(Sung by a renowned sufi singer Kailash Kher)

May the Existence of mine be crushed; may this turn out to be corpuscular. Let me taste the '**taste of completeness'**, let me be complete, let me be ample like you......

For that, please do the ceremony of ashes

Please bestow me the rainbows of existence filled with vibrant celebration.... Oh, Aadi Yogi 'Oh Eternal Existence' Oh "**Master of connectedness**"

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Those, the brains who are a bit interested in natural science and meta science like Mathematics, may surely be trapped at two mystic words; one is *completeness* and other is *connectedness*. Actually, these two are not merely the words, but I think these are the natural laws which have been practiced by the entire universe since inception. If we recall the evolution theories of creatures and development theories of societies from either of the perspective (i.e., either scientific, philosophic, psychologic or Vedic), we observe that the zest of every theory condensates over these two natural laws. Surprisingly, knowingly or unknowingly, each and every constituent of this universe is obeying these kinds of laws.

The intellects who have been or are curious about such mystic laws such as the ancient sheers of Vedas (e.g., Kapil of Samkhya Darshan, Patanjali of Yoga, Kanad of Vaisheshikha etc.) Philosophers of eastern, and western continents (e.g., Immanuel Kant, Hegle, Karl Max, Confucius, Max weber, Gaetano Mosca, Socrates, Plato and Aristotle etc.), Psychiatrist (e.g., Charak, Sigmund Freud, Carl Jung etc.) and scientists (e.g., Darwin, Pythagoras, Archimedes, Isaac Newton, Albert Einstein, Ramanujan, SN Bose etc.), have tried to depict their experience in many ways and have got succeeded in grasping few glimpse of these very natural laws. Really, their work projects their laws of logics for the logic of law.

But what about person like me or probably you...? Though, there are individuals like me, who has been inspiring from something lawlike "*Ocean exists behind the waves*", but what hinders us from having the glimpses of '*logic of law*' and projecting the experience to the fabric of universe? **My answer is- Infinity**

2. LAW OF INFINITY

A prominent thing, that always clicks my mind is that: the natural laws are awful and to dare logics behind these laws is even more awful. Here, by awfulness, I simply mean 'fear of loss', (खो जाने का डर). Fear of loss of what? Of course, fear of loss of own identity "खुद के खो जाने का उर" (which the sheers of Upanishada called False ego). And where might we loss ourselves? Where, there is no dearth of anything that we desire, i.e., a 'domain of completeness' (like you want some water, for that you will never sit in the lap of ocean...). And once, you be immersed in completeness, you would have no kind of desire, you are totally lost... Now your so called 'identity', your 'self' immediately embeds in a mighty self, i.e., 'cosmic conscious domain, a premise of completeness'. Before, propagating further to the theme 'ocean exists behind the waves', let me tell you a very interesting event which often occurs in our inner world of minds, and probably every individual has surely witnessed such an event at least once in his life. The event is 'Law of attachment, attraction, love, affection, compassion etc.' In either of such events, there always exists 'Fear of loss'! When you are in love (Call it domain of completeness), you are lost in what you call 'Indeterminate' or an 'Infinitude of love', that's why the famous wisdom seeker of India known as Kabira called it "A congested/ narrow street of love, where the two cannot be merged"ⁱⁱⁱ, and the lines Mirza Galib (the famous Urdu and Persian poet) 'सुकून और इश्क़, वो भी दोनों एक साथ, रहने दो गालिब, कोई अक्ल की बात करो?", i.e., "Ishq bhi ho aur sukun bhi ho, Suno Janab hosh me to ho" are indicating the same law 'fear of loss'. I personally found no other valid argument in support of 'fear of loss' than the one given by a famous Indian poet Tulsidas who in one of his masterpiece 'Ramcharit-Maanas' mentioned that (बिन भय होत ना प्रीत), i.e., "vou are in love 'if and only if' you feel fear of loss " So, in continuation to the present them, let me talk about that 'Indeterminate' or 'Infinitude' a 'premises of completeness' (whatever you call it) or the source which sinks the individual self, and I guess many of us have been facing it at every stage of life.

Being a Mathematics lover, let me talk about that kind of 'Indeterminate or Infinitude', which I faced throughout my academic life and still facing.... BECAUSE, it's really a frightening as well as an astonishing experience to define *that, which shall not leave you alone*.

A famous Mathematician David Hilbert, in his speech about mathematical problems, on the occasion of his honorary citizenship of *Königsberg*, defined the 'indeterminate' as '*we must know*, *we shall know*'. With this argument of Hilbert, let me introduce you to the content of this concept 'infinitude'. You might have probably seen a symbol representing a number eight in sleeping mode. That is, a lazy 8 symbol: which is technically called 'Lemniscate' and for the first time



in 17th century was used in a treatise of conic sections. Later on, it was devoted to symbolize 'infinity' or 'eternity' in a variety of context, e.g. in 1700s, it began to be used on Tarot cards, known as Juggler or Magus^{vi}. However, for the George Cantor, the founder of modern mathematical theory of infinity, it was cognised to be as a Hebrew letter \aleph (alef). The significance of this symbol lies in the heart of my theme 'Ocean exists behind the waves', where one can travel around this curve with an axiom of choice that either you will know the initial and terminal of this or not...

Regarding this 'indeterminateness' I have already talked that the law of infinity commonly inspires feeling of awe, futility, fear and negligence etc. But, to the Mathematicians, especially those of Greek's admitting the notion like 'infinity' was really horrible dream. They feared infinity and started avoiding it, but this refraining attitude led them the foundation of infinite processes in 19th century calculus^{vii}. On the other hand, George Cantor (1873-1884), a famous German Mathematician and the founder of set theory had proposed a new kind of infinity, which he called 'Uncountable infinity', which he validated by placing law like 'into the continuum' where he talked about infinity times infinity. He ended up his continuum hypothesis by quoting a notion '*To Aleph-Null and Beyond*'', where he had referred the following:

"Either way, we know we have got aleph-null² and (at least) aleph-one, and while they are both infinite, the latter is considerably more infinite than the former. But are those the only infinities? Can we go still further to aleph-two, aleph-three, and so on and so forth? It is indeed possible to take things further, and all we need is one more concept: the power set"."

3. HOW BIG IS THE INFINITY?

Even after Cantor's continuum hypothesis and in fact till 19th century, most of the philosophers and mathematician were still disinclined to admit infinity more than 'potential'. The infinitude of something, some process, assemblage or magnitude was considered as the probability of its indefinite continuation. For instance, the natural numbers 1,2,3,..., can be admitted as a potential infinity- generated from 1 by the process of adding 1- without admitting that there is a completed totality. A more fascinating logic came into existence when a sequence of rational numbers found to be saturated at a certain limit. Then, for Greek's, it became very tempting that how an infinite sequence of numbers is getting nearly closed with a potentially finite number. Anyways, Greeks were feared of drawing such a conclusion, as they were already frightened off by the 'Paradox of Zeno- around 450 BC'^{ix}.

² Aleph-null was to symbolize the set of natural number, and aleph-one was aleph-null+1

Aristotle in his Physics book (Vol 6, Chapter-9)^x quoted Zeno to refute the 'Zeno's Paradox'. He wrote, it is not clear what Zeno wishes to achieve? Was there, for instance, a tendency towards conjecture about the infinity that he disposed of? Anyways, Zeno's arguments were so extreme that almost all the arguments of his contemporaries became strike off.

Let me mention one of his paradox, now known as 'Paradox of motion' or the dichotomy:

'There is no motion, because that which is moved must arrive at the middle (of its course), before it arrives at the end'.

Aristotle, described the full argument of the Zeno as:

Before getting anywhere, one must move half way, and before that a quarter of the way, and before that one eighth of the way, and so on so forth infinitum.

The completion of this infinite sequence of moves does not seem to be impossible for most of the mathematician after indulging the miraculous work of **Srinivas Ramanujan**, a famous Indian Mathematician whose admirable efforts are reflected in a book "*The man, who knew infinity*".^{xi}

Just look at the fantastic numerical work of Ramanujan on sequence and series, which is not merely an act of playing with numbers, but to me it's an exemplary work in connection with "law of logic Vs logic of law". For instance, consider this one:

$$1 + 2 + 3 + 4 + \dots + \infty = ?$$

From a layman perspective the answer should be 'unknown' or infinity. But you would be surprise to know that the sum of this infinitum of natural numbers is going to be -1/12, O my God!!! What a miracle? Sum of the infinitum positive things is not merely a negative quantity but rather a negative rational number. Look at this beauty, it is glorifying the 'completeness of infinitude'. Don't you think now, 'completeness' is the premises of 'infinity' and thus validating the statement that 'law of logic plays in the lap of logic of law'? If not, let's have another one:

The Grandi's Series: $x = 1 - 1 + 1 - 1 + 1 - \dots$

What would you conjecture now? Okay, it dramatically changes its results: By first way, we have the answer x = 0, by second way it reveals x = 1 and finally by third way, it gives x = 1/2. Again, revealing how infinity shifts its spectrum according to the premises of completeness.

At this moment, in addition to the history of infinity delineated as above, I would like to mention the contribution of ancient Indian Mathematicians, like Bhaskarachcharya, a 12th century intellectual, who wrote Algebra (Bijganita) and Lilavati (the book of arithmetic), but unfortunately could not be honoured like Newton and Leibnitz.

In his book of algebra, we can find the concept of infinity (See Pics. 1 and 2)^{xii}



Figure 1: Characteristics of infinity proposed by Bhaskaracharya



Figure 2: CHaracheristics of Zero in Bhaskaracharya's Algebra

4. MANY INFINITIES:

In the previous stanzas we have travelled through many infinities. But the idea of their size is perhaps the most counterintuitive discovery ever made by Mathematicians. Now, in conclusion, I would like to quote a recent research which claims that two different infinities are equal in size^{xiii}.

5. IF INFINITY IS COMPLETE THEN WHAT IS COMPLETENESS: WANDERING WITH COMPLETENESS

Let's have a philosophical as well as mathematical romance with completeness, for that, let me

choose a path proposed in Ishavasya Upanishada^{xiv} (See picture 3^{xv}). What you think? It's a puzzle or really authenticating the statement "Ocean exists behind the waves"? To me, again these rhythmic patterns of logical ideas are unfolding the mystery of 'cosmic completeness'. Let me re-interpret these rhythmic lines in accordance to my instinct:

'That whatever is complete, will ever remain complete Either you add something to it, or subtract anything from it, there never happens a dearth in this complete. Eventually the completeness of the complete remains intact.'



Figure 3 Logic of law for Completeness

Look at this miraculous stanza! Surprisingly, to me this is a consistent logic of natural law. More precisely, this is the law radiating the criterion of 'Being Complete'. If we meticulously go through the development of number theory, we find that the property of being complete has been a most fascinating idea associated with real number system. In fact, in an intuitive way, completeness implies that 'now there are no more holes or gaps in the real number system. Richard Dedekind (1831-195), a German mathematician has been one of the intellectual, who for the first time contributed a systematic logic for completeness. In his terminology for axiomatic foundation of natural numbers, he evoked that

"completeness in the sense of Dedekind, is the property that-*Every Dedekind cut/ hole of the real numbers is generated by a real number*^{,,xvi}

In addition to this, mathematical completeness is equivalent to the statement that 'an infinite string of decimal digits is indeed a decimal depiction of some real number'. However, numerous versions of completeness axiom were derived during the discovery of number systems. Apart from Dedekind completeness property, many other axiomatic approaches including 'least upper bound property', 'Cauchy completeness axiom', 'Nested interval theorem', 'Intermediate value theorem', 'Monotone convergence theorem' and 'Bolzano-Weierstrass theorem' etc. were developed and these altogether became a package for completeness. However, among all these, 'Dedekind completeness' and 'Cauchy Completeness' criteria were prominently considered to be the gems of completeness.

We have talked about Dedekind completeness, which talks about the Dedekind cuts or holes. Let's, have a look at-what the renowned French Mathematician 'Augustin Louis Cauchy (1789-1857)' thought about completeness? In his terminology:

The Cauchy completeness is a kind of a logic like -

"Every Cauchy Sequence of real number converges"xvii

Those who are familiar to the content of Mathematics, may be aware of the notion of convergence of sequence, however the picture (see pic 4^{xviii}) placed here revealing all the 'analogies of completeness' from Ishavasya Upanishadic standpoint to Cauchy's standpoint. To me this picture reveals a lot. It suits best to Upanishadic completeness, where addition and subtraction of things to this pictographic structure does not change the harmony of structure. Also, from Dedekind perspective, the Dedekind cuts or holes in this structure are generated by the constituent elements (the real numbers) of the structure. If we keenly observe this pictorial structure, we can incorporate the Cauchy's criterion of completeness, which delineates that-

the elements of this infinitude mathematical construct are tending closer and close while travelling along the positive sense of the structure.

6. TRANSCENDING INCOMPLETENESS LAW-CAN WE CONSTRUCT A COMPLETENESS FROM **INCOMPLETENESS?**

At this end, I would say 'Yes' of course! In fact, the approaches of defining and understanding completeness discussed in the forgoing stanzas are nothing but the devices developed to transcend the incompleteness, either Figure 4 Analogies of completeness

it be metaphysical or physical. It may seem to be incredible

to hear, but either of the above devices stand for seamless experience of constructing a complete out of incomplete. The whole approach of almost all famous number theorists was to discover devices, that could handle the law of 'Transcending incompleteness', for instance, constructing Rational number system out of Integer system, and then constructing Real number system out of Rational one.

We are fortunate to have an adequate number of good candidates for this law of transcending incompleteness. These include the David Hilbert's grand hotel paradox, the infinite pigeonhole principle, the travelling salesman problem, the Tower and Hanoi puzzle, The infinite chocolate paradox, and the last but the prominent one- the Banach Tarski paradox.

Let's take few glimpses of what Banach-Tarski paradox states about...



The Banach-Tarski Paradox- It's a well-established theorem in set theoretic geometry and describes that "For a solid ball in threedimensional space, there always exists a device called decomposition, which can decompose a given ball into a finite number of disjoint point sets, which can be pulled back to reconstruct the two balls identical to the original one". However, the reason behind calling this theorem a 'paradox' is that it contradicts the fundamental intuitions of Geometry (see picture 5^{xix})

We have another candidate "Infinite Chocolate Paradox", which is extracted from Banach-Tarski Paradox and thus leads us to similar law of 'Transcending incomplete'



Figure 5 Banach-Tarski paradox- making infinite copies from a finite

I admit that this 'infinite chocolate paradox' is

also telling us the same truth, which the rhythms (Fig. 3) of Upanishada have been told. So, enjoy the taste of a neverending chocolate (see pic 6^{xx})



t towards the e waves'. *Figure 6 Infinite chocolate paradoxtaste of never-ending Choco bar*

After what have been discussed up to this extent, it seems necessarily logical to me, to catalyse my intellect towards the very essence of my theme 'ocean exists behind the waves'.

That very essence is none other than '*the law of mastering the connectedness/ Unity*' which is of course, a sensor of completeness.

7. LOGICAL LAW OF CONNECTEDNESS OR THE UNITY-

To me, state of being seamless or conjunct or affinity or more precisely, state of being compassionate has always been a device of joying the completeness. During high-school classes of Biology, I, for the first time heard about ecological systems at the substratum of whose, there is nested an intrinsic law of food chain. At that time, I never thought about the significance of ecological systems. Now the things become much clearer to me, and I can experience the pain of ecologists, whose intellect dances in a chaotic way, when frog (for instance) disappears from the law of food chain. I guess, you too might be thinking of conserving the consistency of the law of connectedness, so that the entropy of completeness could rest in equilibrium.

Apart from the ecological system notion, these are some more subtle notions over connectedness described in one of the ancient India school systems, known as '*The Vaisheshika System*^{xxi}', founded by '*Kanada*'. Let's take care of the sutras of Kanada, which are naturally interpolating the laws of being connected in terms of constituent entities of the universe.

2. The second logic of Kanada states the properties possessed by earth, i.e., 'Earth retains the characteristics or Tanmatras, namely-Colour, Taste, Smell, and Touch'



- 4. The fourth sutra is "Fire possesses colour or luminousness and Touch/ Heat"
- 5. The fifth sutra states that "Air possesses Touch"



वती प्रथिवी ॥ २ । १ । १ ॥

आपो द्रवाः स्निग्धाः ॥ २ । १ । २ ॥

6. And the sixth verse says that "None of the Tanmatras discussed in previous five sutra are possessed by Space or Aether".

What a mesmerizing idea of Kanada!!! The idea (1-to 6) has blended down all the ambiguities of my theme. I don't know why *Kurt Gödel* had not gone through the Vaisheshika Sutra of Kanada? I wish, May *Gödel* would have been gone through these sutras of completeness and connectedness.

Anyways, my contention is that- Like an ecologist, an alchemist will surely feel compassionate towards the connectedness of gross elements as well as the gross universe. It is a matter of fact that, if somehow, we could remove the Tanmatra like 'smell' from earth, we are surely going to have another gross element called Water. Again, if we somehow detach the Tanmatra called 'Taste' then this leads us to get another gross element 'Fire' and we can continue the process of decomposition until we achieve an element called 'aether or space'. So nice to see that "*How a complete entity is producing another complete entity*"

At this corner, I have to say that in an Indian masterpiece "Mahabharata", I have gone through a story, where an act of an alchemist known as "Durvasa" surprised me. In that story, what he did is that, he took water on his palm and spread it on earth, and suddenly the fire appeared at that place. May be, he would have been well versed with the law of connectedness.

8. HOW TO BE A MASTER OF CONNECTEDNESS?

From the standpoint of a Mathematician like Erwin Kreyszig, a renowned author of functional analysis, "Any domain D is called simply connected, if every closed curve in D can be continuously shrunk to a point in D, without leaving D" (see Fig. 7)^{xxii}. However, there are more complex forms of connectedness, such as doubly connected, triply connected, N-ply connected and multiply connected mathematical structures (See Fig. 8, 9)^{xxiii}



Figure 7 Simply connected and non-simply connected circuits



Figure 8 Simply, doubly and triply connected circuits

A more fundamental definition from the account of set theorist states that: "A set is said to be connected if any two of its points can be joined by broken line segments, i.e., by means of a polygon"

My sincere greetings go to Henri Poincare, who at the very beginning proposed his famous conjecture for connectedness - "Every simply connected closed three manifold is holomorphic to the 3-sphere", whose valid proof was presented by Grigori Perlman (2002-2003). Finally,

thanks to Richard S. Hamilton, who shared his mastery over the connectedness of mathematical domain. What he did, has been a milestone and a gift for entire mankind. He had devised a device now famously known as "Ricci flow with surgery" (See Fig. 9) ^{xxiv}. And



Figure 9 Ricci Flow with Surgery- turn multiply connectedness into simply connectedness

this device is presently adopted by whole scientific community to systematically tackle the singularities found in multiply connected domains.

At this end, I would like to sum up my theme with a quest about-*Who is the master of Unity or connectedness*?

To me- the one who has an ability of transcending incompleteness is a master of unity...

What about you???

I am leaving you alone in this expedition with few clues given below:

There is an amazing verse quoted in the Patanjali's Yoga sutra (See Pic. 10^{xxv}), which is indicating the right path of this quest.

Another clue- From Bhagvat Gita (Pic. 11^{xxvi}):



Figure 11 Who is Yogi (or the master of unity)

<u>Transliteration:</u> paramāņu paramamahattvānto'sya vaśīkāraḥı <u>English translation:</u> The mastery over mind results in control of relationship with everything from atom to cosmos Figure 10 Who is the master of connectedness

In this verse, Lord Krishna is giving clues about Yogi (the master of unity) to Arjuna, with regards to the query "*Who is Muni?* "posed by Arjuna. And then

Krishna is replying- The one, who not merely thinks about fruits of Trees, but also thinks about tree-hood-ness, who not merely thinks about water, but also thinks about river-hood-ness, who not merely thinks about human, but also thinks about man-hood-ness. In all, that who is compassionate for nature and always cognizes the completeness of the mighty cosmos- is the one that I call 'him Yogi'

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